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Social sciences: a third level course

COGNITIVE PSYCHOLOGY

BLOCK 4

LEARNING AND PROBLEM SOLVING (part 3)

Unit 28 Formal analyses of problem solving behaviour

Part 1 (of 3)

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STUDY GUIDE

This unit discusses some newly-developed techniques for analysing human problem solving behaviour. These techniques are related to some of the representational systems developed in Units 26–27, but are developed with an eye towards accounting for experimental data. The unit is divided into two parts. In the first part, I discuss the way in which the *structure* of a problem can influence the problem solving behaviour of groups of subjects. The structure of a problem is itself revealed by looking at a 'tree' of possible moves, comparable to the 'trees' you saw for the missionaries and cannibals and Fifteen Puzzle problems in Units 26–27. These trees, or 'state-spaces', as I shall call them, can serve as an ideal skeleton upon which to build up a detailed account of a subject's behaviour.

In the second part of the unit, I discuss a new modelling tool which is gaining increasing popularity in cognitive psychology and artificial intelligence. This tool is a new style of programming language known as *production systems*. Deceptively simple in design, production systems offer a kind of flexibility which is very difficult to obtain in standard step-by-step programming languages like SOLO. One of the main motivations for adopting these production systems is the ease with which they can be used to simulate in detail the actual steps which a subject takes in the course of solving a problem. The subject's analysis of a problem will be laid out in a format known as a 'problem behaviour graph', very similar to the standard 'trees' you are already familiar with from Units 26–27, except that in this case each node of the tree will represent the subject's own 'mental state' rather than the state of pieces on the board. The Appendix starting on p 103 contains two applications of production systems to problem solving tasks. The first is the Tower of Brahma which you have already tried. The other is children's behaviour when learning to put blocks in size order (seriation task). The *entire* Appendix is not set reading but you will be told when to read it and in how much detail. The seriation task will also be discussed in detail in TV 14, so you will find it useful to have read this section of the Appendix as background reading.

Reading for the unit

Optional reading for this unit will be two articles in the Johnson-Laird and Wason set book, *Thinking: readings in cognitive science*. The first is by Allen Newell, one of the prime movers (along with Herbert Simon) in the development of production systems and problem behaviour graphs as psychological modelling tools. In the article, Newell describes in detail the analysis of the behaviour of one subject solving a difficult problem. The second article, by Terry Winograd, compares and contrasts several different representational formalisms, including semantic networks and production systems. The article may help to clarify some of the difficult interrelations among different styles of representation, each of which is typically (and unfortunately) geared for tackling a particular kind of problem!

Objectives

By the time you finish reading this unit, you should be able to:

1 define a *state space* for a problem or game, including *start* and *goal states*, sub-problems and sub-goal states, and sets of legal moves;

2 define and give examples of problem *isomorphisms*;

3 describe the use of a state space analysis to study the behaviour of subjects solving the Tower of Brahma and Parking Lot problems;

4 describe the use of the state space technique to study the effects of *transfer* and changes in *task instructions*;

5 describe a *problem behaviour graph* and show how it is constructed from a verbal *protocol*;

6 describe a *production system* programming language and its use as a model of short term memory and long term memory;

7 describe the use of production systems to analyse and compare strategies in solutions of the Tower of Brahma problem;

8 describe the use of production rules to model developmental aspects of seriation tasks.

1 STATE SPACE ANALYSES

In Units 26–27 you saw that the moves one makes while solving a particular problem can be depicted in the form of a 'tree' of move possibilities. This was illustrated for the missionaries and cannibals and Fifteen Puzzle problems among others. Since each node of a tree represents a particular 'state' of the problem, a full-blown tree itself is often referred to as 'state space' (i.e. a large collection of interconnected states).

In this section I will show how these spaces can be used not only as a convenient way of representing a problem inside a computer, but also as a tool which can help us formulate hypotheses about the way in which *people* solve certain kinds of problems.

1.1 Tower of Brahma and Parking Lot problems

Two of the problems we shall be looking at are included in your home games kit. You should do both of the activities below before reading on.

ACTIVITY 1 — 15 minutes

Take the Tower of Brahma from your games kit. Arrange the rings on the left-hand peg (as you are facing the pegs), so that the largest ring (yellow) is on the bottom, with the green ring on top of that, then the blue ring, then the red ring as shown in Figure 1. Remove the fifth ring as you won't be using it.

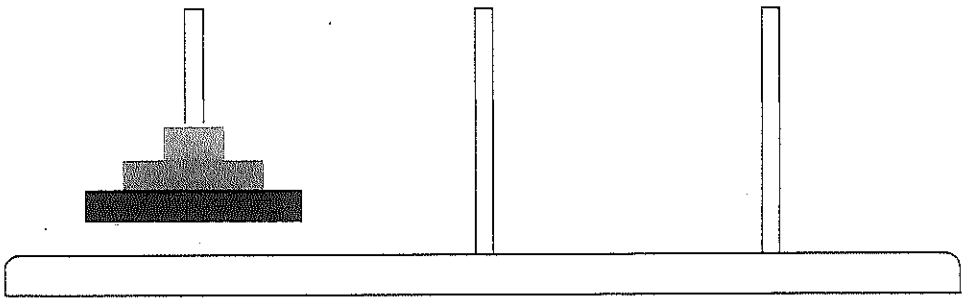


Figure 1
Tower of Brahma problem

Now, *moving one ring at a time*, and one ring only, try to move the configuration of four rings over to the far right-hand peg, so that the rings end up in the same order (yellow on the bottom, etc.). *You may not place any ring on top of a smaller ring*. And, of course, rings may only be placed on one of the three pegs, not placed elsewhere on your table or floor, etc.

SAQ 1 — YOUR ANSWER (MY ANSWERS ARE ON p 124)

What was the fewest number of moves you needed to switch the four rings? Do you think this is the minimum number?

SAQ 2 — YOUR ANSWER

Now use the same *start* position (left-hand peg) and transfer all the rings (by the same rules) to the *middle* peg. What was the fewest number of moves you needed? The minimum solutions for SAQs 1 and 2 are said to be *symmetric*. In what sense do you think this would be so?

SAQ 3

Try the Tower of Brahma problem with only three rings (green, blue, red). Starting with the rings on the left-hand peg, move the three rings (by the legal moves) to the middle peg. This is said to be a *subproblem* of the original one you solved. In what sense is this so?

YOUR ANSWER

---

15 minutes ACTIVITY 2

Take the Parking Lot problem from your home kit (four small plastic 'trucks' and a board showing a Y-shaped pattern of coloured 'parking places'). Place the four trucks on road A (the bottom of the Y-shape) on the squares which correspond to the colours of the trucks (red truck on red square, etc.). Imagine now that trucks from four delivery companies (red, blue, green, yellow) have been travelling into a city on road A, and that they have been travelling in the order shown in Figure 2 (red, blue, green, yellow). As they get into town they find that they

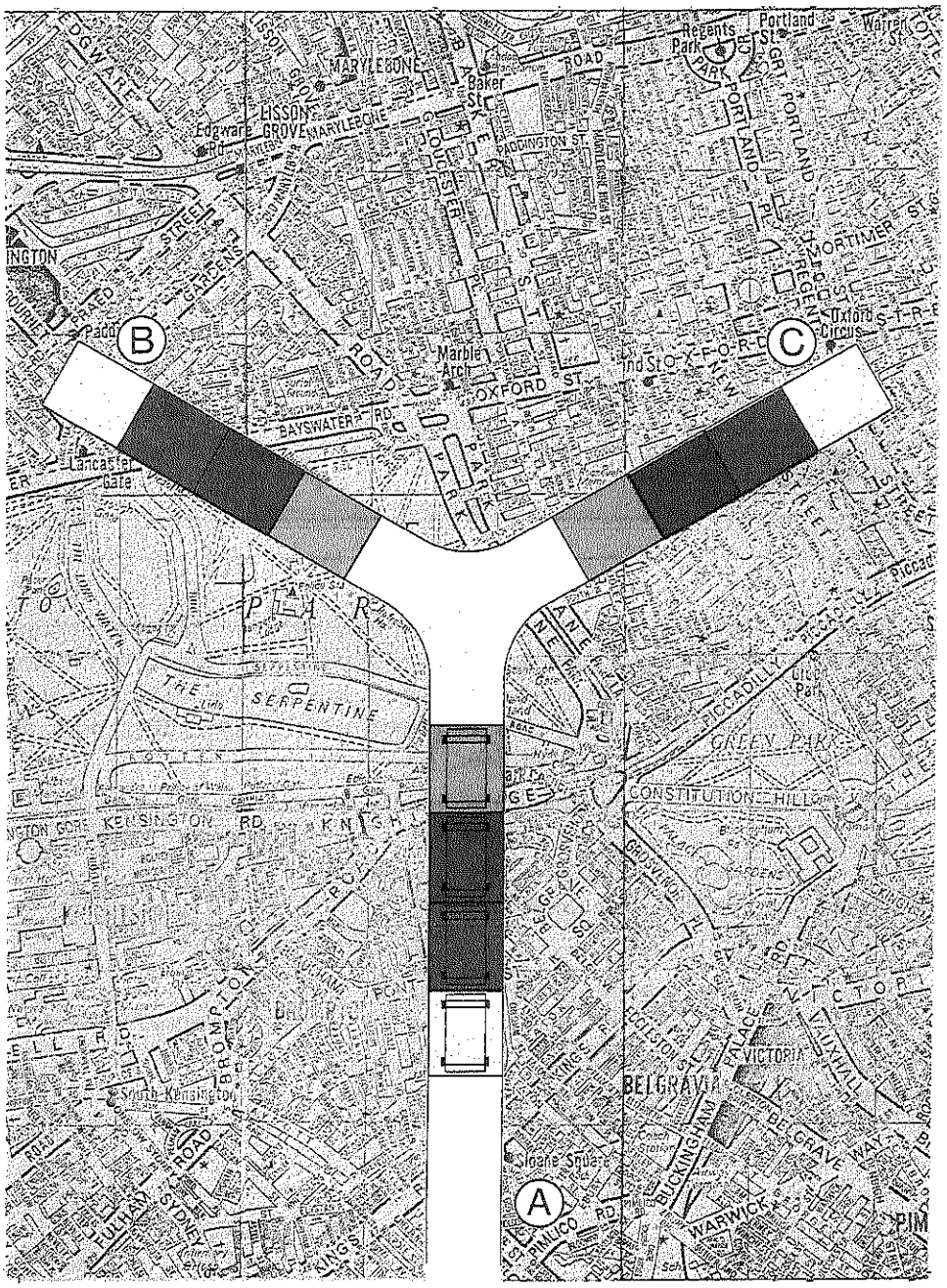


Figure 2
Parking Lot problem

must park in parking places on street C which, to simplify the loading and unloading of cargo, have been allocated specifically for different trucks (thus, the red truck *must* park in the red square, etc.). The streets are narrow, so only one truck may manoeuvre at a time, and trucks can not overtake one another. Moreover, trucks may not pause temporarily in the central (cross-hatched) area, which is watched very closely by traffic wardens. An additional constraint is that each of the four companies owning the parking spaces jealously guards its spaces for its own trucks. Thus, the blue company, for instance, allows rival trucks (red, green, and yellow), to pass over its own (blue) parking space, but it forbids rival trucks to stop *even momentarily* in a blue parking space, as this might lead to lost revenues. Similarly, each of the other companies jealously guards its own parking space, forbidding rivals to stop in it.

Given these constraints, try moving the trucks so that they end up on road C in their proper parking spaces. Remember: a blue truck may only stop on a blue space; stopping anywhere else, even momentarily, is forbidden (and similarly for yellow trucks and yellow spaces, etc.). No overtaking is allowed, as the roads are too narrow! Trucks may move forwards or backwards, and can be moved back to their original positions if necessary.

SAQ 4 YOUR ANSWER

What is the minimal number of moves needed to solve the Parking Lot problem?

You may have felt that the above problems were highly similar to one another, or even different 'variations' of the same underlying problem. If so, you were right. But what is this 'underlying problem'? Is there any way to talk about the relationship between the two problems other than saying that they intuitively 'correspond' to one another? We can, in fact, be *very* precise about the correspondence between these two problems. This precision, as you may have suspected, is brought about by drawing the complete tree of move possibilities and states for both problems, and comparing the resulting state spaces. Here is how we would draw the state space for the Tower of Brahma problem. We would begin with the *initial* (or *start*) state, and show each of the possible states which could result from a legal move. Figure 3 shows how the tree would develop.

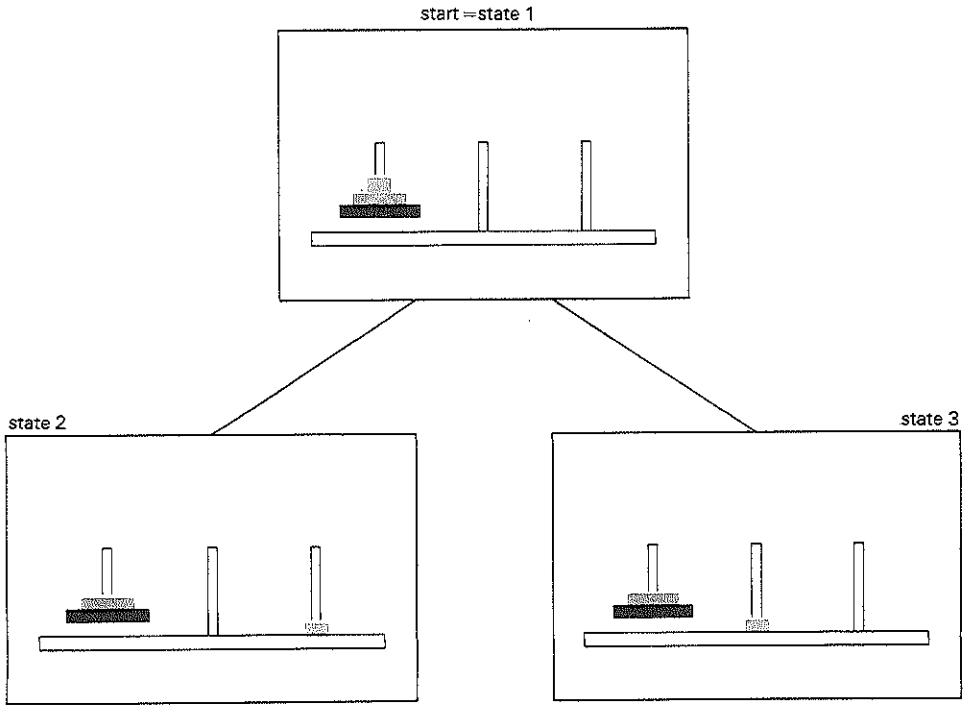


Figure 3 State space diagram depicting initial move possibilities for Tower of Brahma problem

Thus, from state 1, we could go to either state 2 or state 3, depending upon where we decided to put the smallest ring. To avoid cluttering the diagram, I haven't labelled the branches with the actual *moves* (i.e. 'Red ring from left peg to centre peg' and so on). Now, at state 2, there are three possible legal moves: (a) move the smallest ring (red) back to its original place (i.e. go back from state 2 to state 1); (b) move the red ring from the right-most peg to the centre peg (i.e. go from state 2 to state 3); (c) move the second smallest ring (blue) to the centre peg (i.e. go from state 2 to state 4, as shown in Figure 4). So, the three lines emanating from state 2 represent the three legal move possibilities at that point. Notice that a line can always be traversed in both directions, so that we can go from state 3 back to state 2, or from state 3 back to state 1 etc. (This was also true of the lines connecting states in the missionaries and cannibals problem discussed in Units 26–27.)

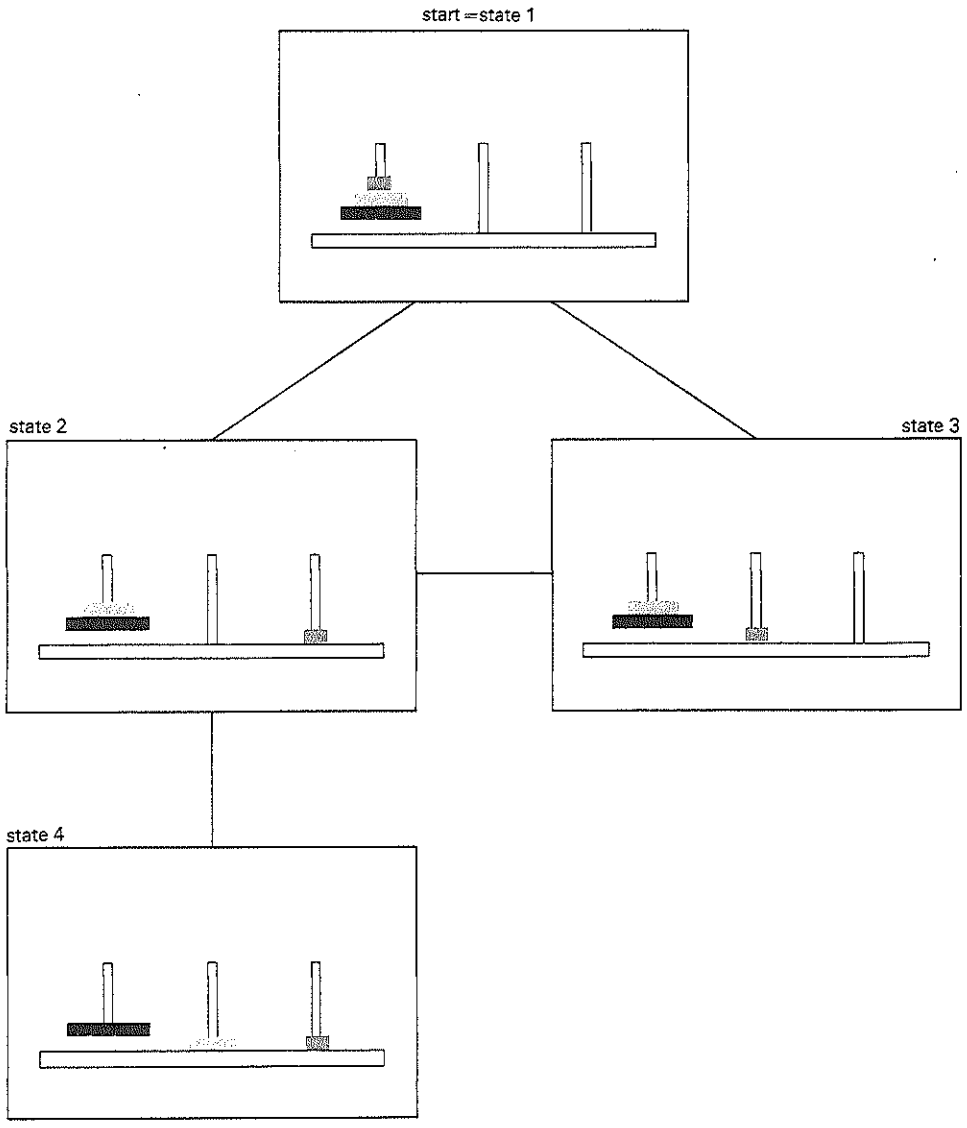


Figure 4 State space diagram showing three move possibilities available from state 2

SAQ 5

On a sheet of paper, draw the entire state space for the version of Tower of Brahma in which you moved only *three rings*, i.e. the red, blue, and green rings to the middle peg. As there are twenty-seven states altogether, you may want to turn to look at the answer after drawing six or seven states yourself.

Rather than drawing a picture to depict each state, we can represent each state symbolically by labelling the pegs A, B, and C (left to right), and labelling the four rings R, B, G, and Y, for the colours red, blue, green and yellow. Then state 1 of the Tower of Brahma could be represented as follows:

A: Y, G, B, R
B: –
C: –

State 2, in which the red ring has been moved to the right peg, could be represented like this:

A: Y, G, B
B: –
C: R

This same notation can be used to depict states in the Parking Lot problem as well, with A, B, and C standing for the roads and R, B, G and Y for the red, blue, green and yellow trucks.

SAQ 6

Using the above notation, draw the entire state space for a version of the Parking Lot problem in which only the red, blue and green trucks are used, and in which these trucks are moved from road A to road B. Compare this state space with that for SAQ 5. (Once again, you may just want to try drawing the first six or seven states before turning to the answer.)

YOUR ANSWER

When we use the same notation to depict states in the Tower of Brahma and Parking Lot problems, we see that the two state spaces are *identical*! Only the physical details of the problems (trucks or rings etc.) are different.

When two problems can be represented by the same state space, these two problems are said to be *isomorphic*. Thus, state space representations allow us to compare different problems, to see if there is a correspondence in the underlying structure of the problems. Moreover, we can study the problem solving behaviour of subjects by analysing the paths which they traverse through the state space of a problem. Paths through problems of related structures can be compared and analysed to see to what extent the basic problem structure affects subjects' behaviour and to what extent differences emerge due to the different superficial details of the problem.

1.2 Problems and subproblems

The Tower of Brahma/Parking Lot problem can be broken down into subproblems. For example, to solve the three-ring Tower of Brahma problem (red, blue, green), it is necessary at some point to move the largest of the three rings (green) from its original position on peg A to peg B. But before this can be done the two smaller rings must be assembled in their proper order on peg C. The problem of moving the two rings from one peg to another may be called a two-ring subproblem, and constitutes a natural subpart of the state space of the three-ring problem. This subpart is simply called a 'subspace', and is illustrated in Figure 5 below. The entire figure depicts the complete state space for the three-ring problem. Notice that state 27 is the solution, with all three rings on peg B. But in the course of this solution, a two-ring subproblem was first encountered, in which it was necessary to get the two smallest rings from peg A (at state 1) to

peg C (at state 9), that is, to get them out of the way so that the green ring could be moved directly from peg A to peg B. This particular two-ring subproblem (i.e. getting both the blue ring and red ring from peg A to peg C) is represented by the subspace which is shaded in Figure 5 (i.e. the 'mini' state space consisting of just states 1 through 9, and ignoring all the rest). The 'finish' state of a subspace (e.g. state 9) is simply called a subgoal.

Notice that, once the green ring is moved across from peg A to peg B (this is the move taking us from state 9 to state 19 in Figure 5), there's another two-ring subproblem to be solved: this is the subproblem of moving the two smallest rings from peg C (state 19), to peg B (state 27). Thus, states 19 to 27 actually constitute another two-ring subspace. Similarly, states 10 to 18 constitute a third two-ring subspace (state 15 is the state in which all three rings have been reassembled on the wrong peg!). So the three-ring state space of Figure 5 consists of three two-ring subspaces. If we were to look at the state space for the *four*-ring Tower of Brahma problem (which has 81 states), we would see that it consists of three three-ring subspaces. By the way, each two ring subspace can be further broken down into three (trivial) one-ring subspaces, comprising only three states apiece (representing the possible moves of one particular ring). A subproblem, then, is depicted by a subspace with its own 'start' and 'finish' states.

There is *symmetry* within the Tower of Brahma/Parking Lot problems. For example, in attempting to move the four trucks from road A to road C, problem solvers often find themselves inadvertently transferring all the trucks to road B. Actually, the pattern of moves to B is the same as those to C, except that B and C are interchanged in the state space representation at every move. What often

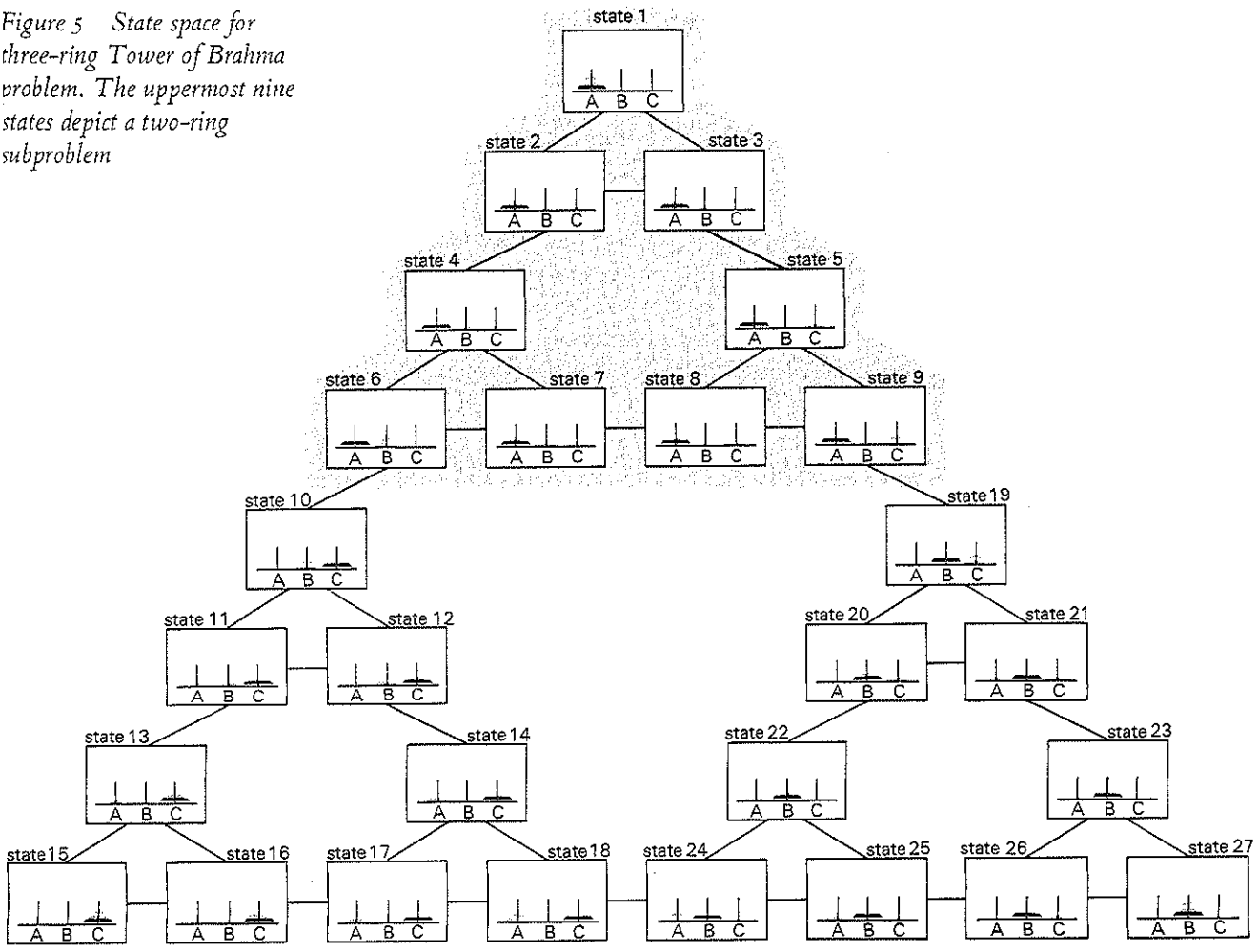


Figure 5 State space for three-ring Tower of Brahma problem. The uppermost nine states depict a two-ring subproblem

happens, in fact, is that problem solvers learn the correct pattern of moves even when going towards the wrong goal! The reason for this is that the *pattern* of moves in each case is identical.

To see how a state space is employed in analysing behaviour, let's take a close look at a two-ring subproblem. Set up your Tower of Brahma in its three-ring initial state (green, blue, and red rings on the left-hand peg, which we call peg A). Now carry out the following five moves:

1 red ring to peg C (right peg);

2 blue ring to peg B (middle);

3 red ring to peg A (left);

4 blue ring to peg C (right);

5 red ring to peg C (right)

Let's look at these moves in terms of the *path* they follow through the state space. We only need to look at the uppermost nine states shown in Figure 5, because we are only interested in the movement of the first two rings (hence, a two-ring subproblem). These states are depicted in Figure 6 (showing actual drawings of the rings and pegs rather than our symbolic abbreviation for the states, just to make the diagrams easier to follow). The same analysis would apply to the Parking Lot problem, since we have already shown that the state spaces are identical. Notice that the pathway made by our five moves is shown by extra-thick lines in Figure 6. This is not the shortest path from the start state to state 9. The shortest path has three moves through the subproblem space.

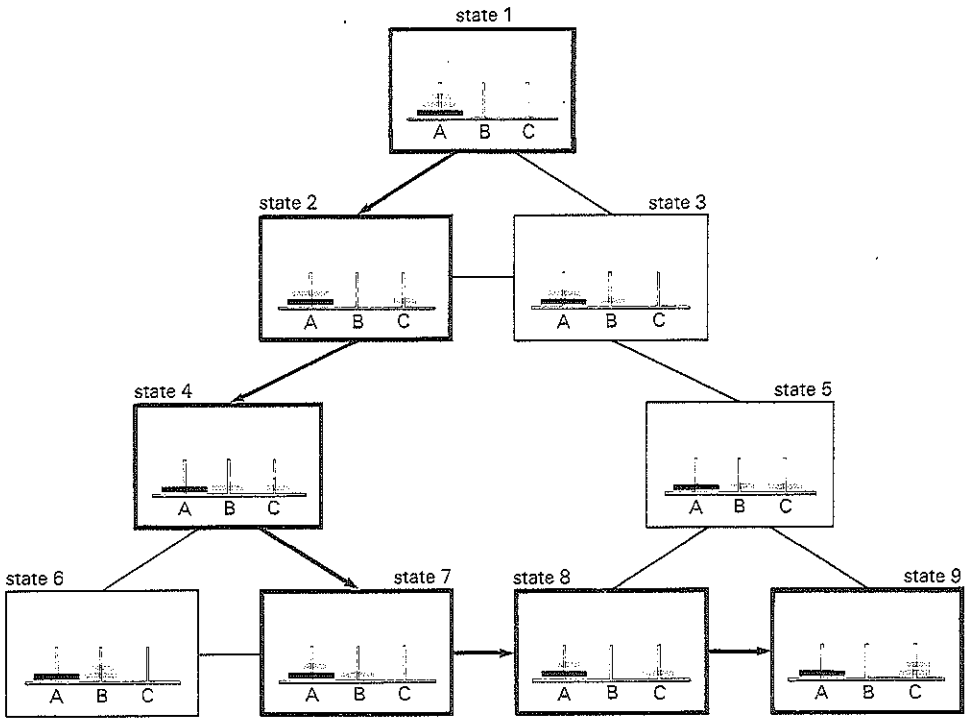


Figure 6 A five-move pathway through a two-ring subproblem space

SAQ 7

Draw the path of the three-move solution to the subproblem considered above. What are the actual moves involved?

SAQ 8

Draw the following five-move path through the subproblem space shown in Figure 7 (beginning at state 9):

1 red ring from C to B;

2 blue ring from C to A;

3 red ring from B to C;

4 blue ring from A to B;

5 red ring from C to B.

Compare this path to the five-move path through the two-ring subproblem shown in Figure 6 which is reproduced below. These two paths are said to be *congruent*. In what sense is this so?

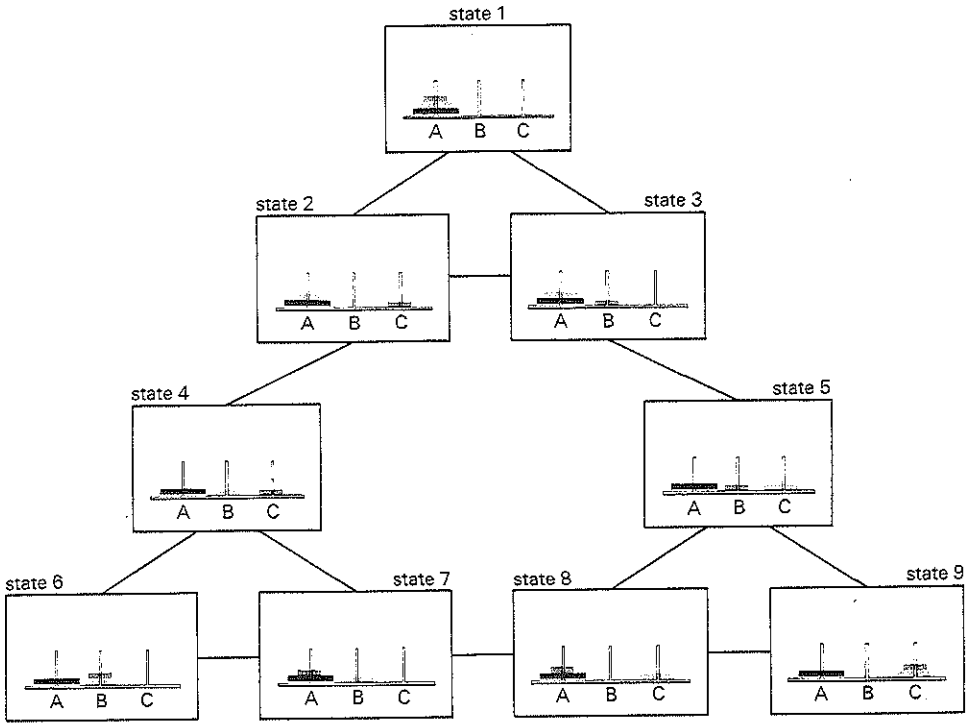
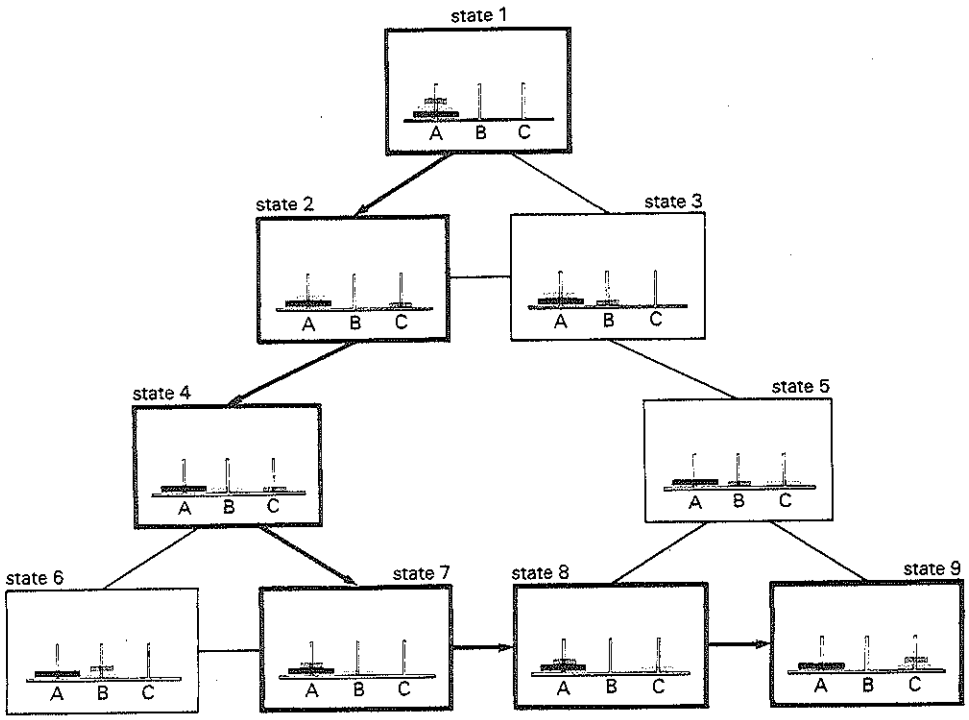


Figure 7 A two-ring subproblem space

Hint: Turn Figure 7 sideways so that state 9 is at the top of the page. Are all the three-move minimum paths through these subspaces also *congruent* (with one another, that is)?



1.3 The path of a subject through a state space

Any subject trying to solve the four-ring Tower of Brahma or the four-truck Parking Lot problem will inevitably take some path through the complete 81-state state space (as this defines all possible states a subject can reach). The behaviour of the subject which is available to our observation consists of the particular path he or she happens to take and the amount of time needed for each move from one state to another. If we focus on the formal properties of the state space, we can begin to think about certain aspects of the problem *structure* which have an effect on the observable behaviour of a subject. In my own research (e.g. Goldin and Luger 1975; Luger 1976), this emphasis led to the formulation of the following kinds of hypotheses (note that these hypotheses are not mutually exclusive, but rather represent a body of independent predictions based on the structure of the Tower of Brahma/Parking Lot state space):

Hypothesis 1

In solving a problem (or subproblem) the subject follows a *goal-directed* path in the state space representation of the problem (or subproblem).

Hypothesis 2

Whenever a subgoal state is reached, the path exits from the subspace of the just-completed subproblem. For example, in Figure 5 on p 68, once state 9 has been reached the path of the problem solver should go on to state 19, rather than states 5 or 8, which are both still within the subspace of the just-completed two-ring subproblem.

Hypothesis 3

Identifiable 'stages' occur during problem solving correspondence to the solution of various subproblems. That is, path segments occur during problem solving which do not make up the direct solution of a problem, but which are direct solutions to subproblems.

Hypothesis 4

The problem solver's paths through isomorphic subproblems tend to be congruent (i.e., the same shape, as discussed in the answer to SAQ 8).

Hypothesis 5

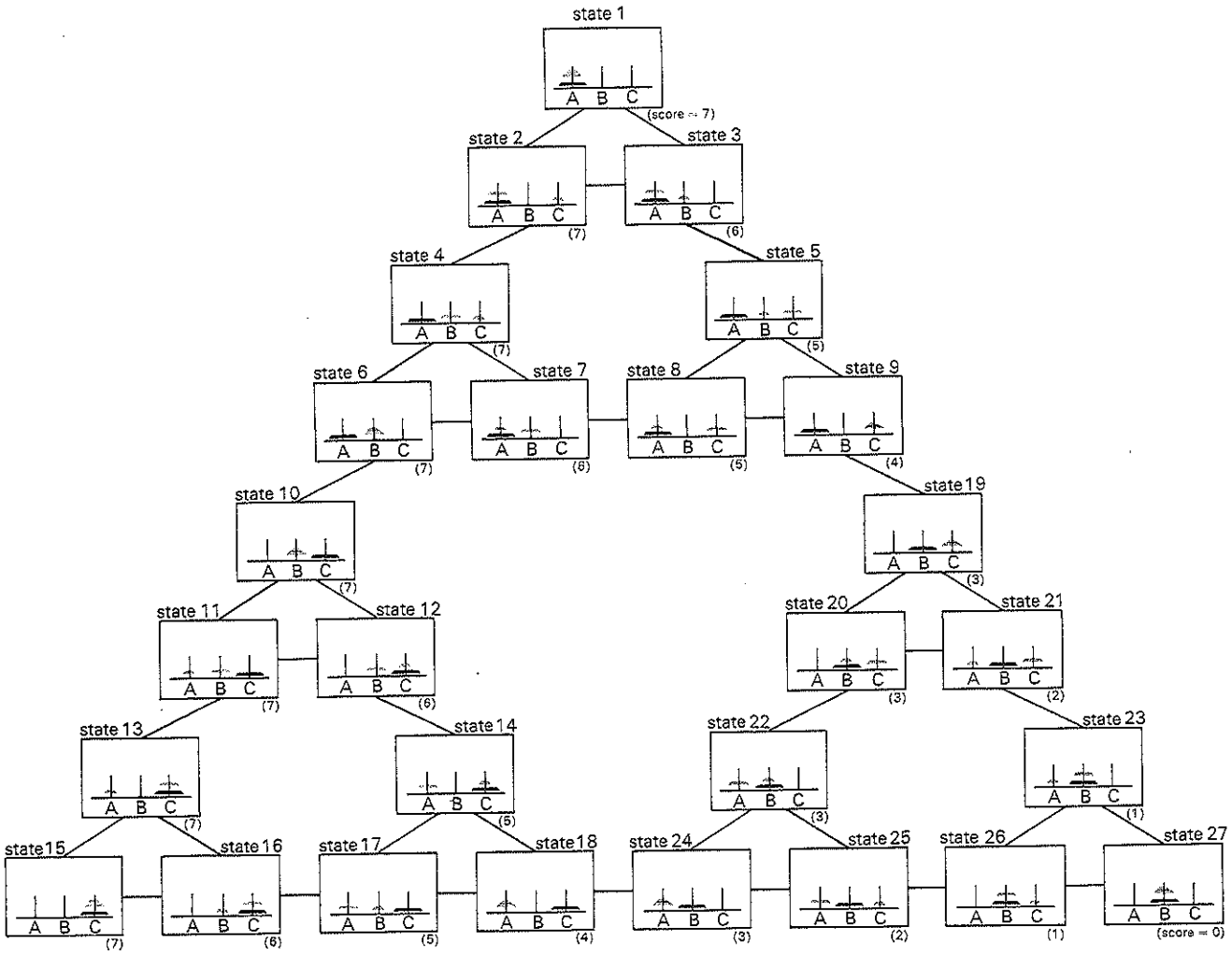
When symmetries exist within the state space of a problem (as discussed on p 68), subjects tend to produce successive path segments that are symmetric to each other (i.e., a mirror image path).

These hypotheses are not an exhaustive list, but indicate the kind of analysis possible of the effects of problem structure on the problem solver's behaviour. Let's take a more detailed look at how these hypotheses can be tested.

Hypothesis 1 spoke about goal-directed paths. In terms of the state space representation, we can say that a path is goal-directed as long as there are no moves along the path which take the problem solver from one state to another which is further from the goal (i.e. as long as no 'harmful' moves occur). We can determine this by giving each state a *score* which reflects its shortest distance (in number of moves) from the final goal (or subgoal, if we are looking only at a subproblem). These scores are illustrated below for the three-ring subproblem you solved for SAQ 3, i.e. moving the rings to the middle peg. The score of 5 for

state 8, for instance, means that you can get from state 8 to the goal (state 27) in 5 moves *at best*. Obviously, more circuitous pathways might require more moves.

Figure 8 A three-ring subproblem space, showing scores for each state (i.e. distance from goal state 27)



SAQ 9

The scores shown above are very similar to the 'evaluation function' scores for the Fifteen Puzzle discussed in Units 26–27 section 3.3. There is, however, one vital difference. Can you see what this difference is? (*Hint:* compare the way in which scores are calculated for the states in the Fifteen Puzzle with the way in which scores are calculated for states in the Tower of Brahma puzzle.)

YOUR ANSWER

Opposite are two pathways made by two subjects solving the three-ring Tower of Brahma problem:
The successive distances from the goal (state 27) are:

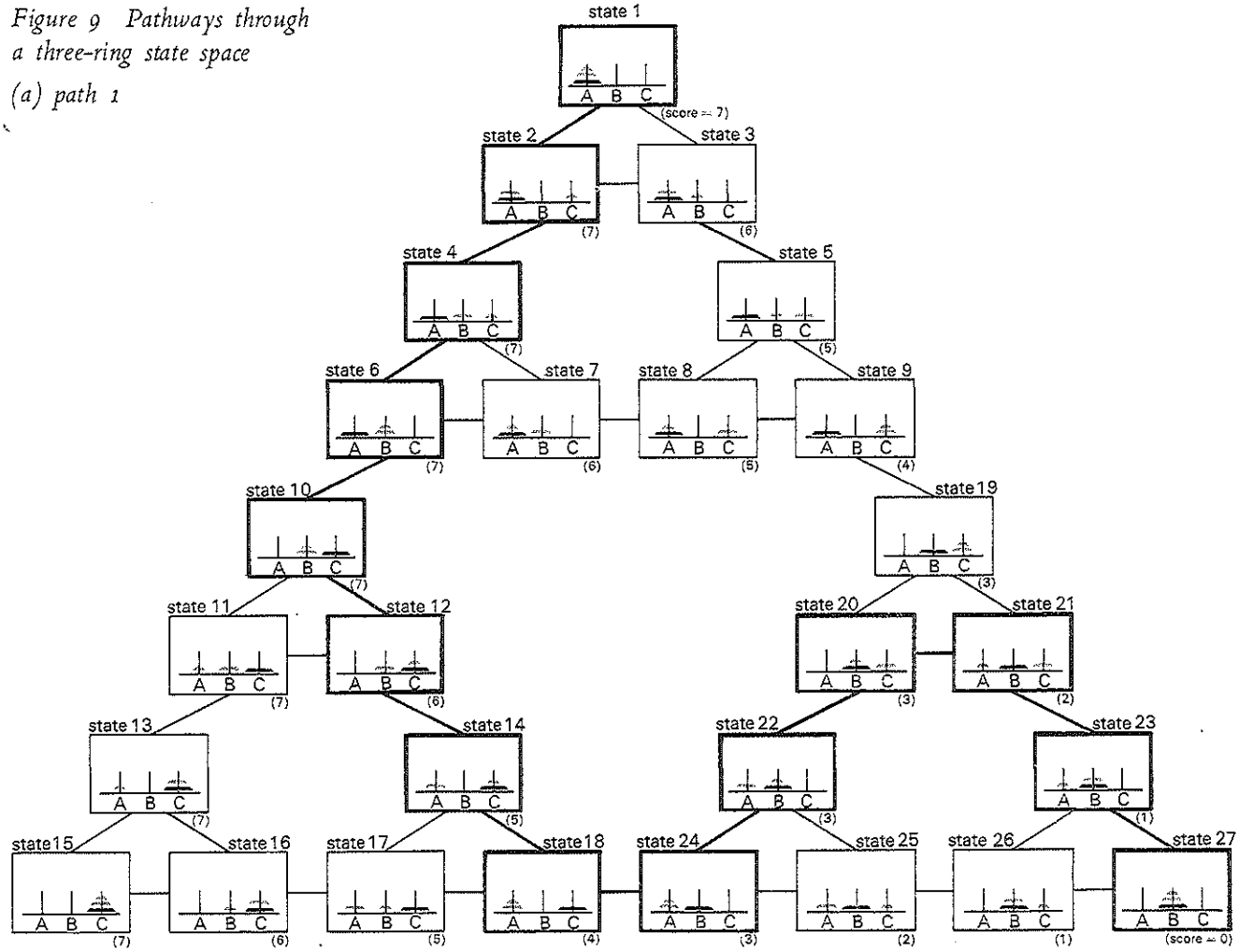
for path 1 (shown by thick lines in Figure 9a): 7, 7, 7, 7, 7, 6, 5, 4, 3, 3, 3, 2, 1, 0;

for path 2 (in Figure 9b): 7, 6, 5, 5, 6, 7, 7, 7, 7, 6, 5, 4, 3, 2, 1, 0.

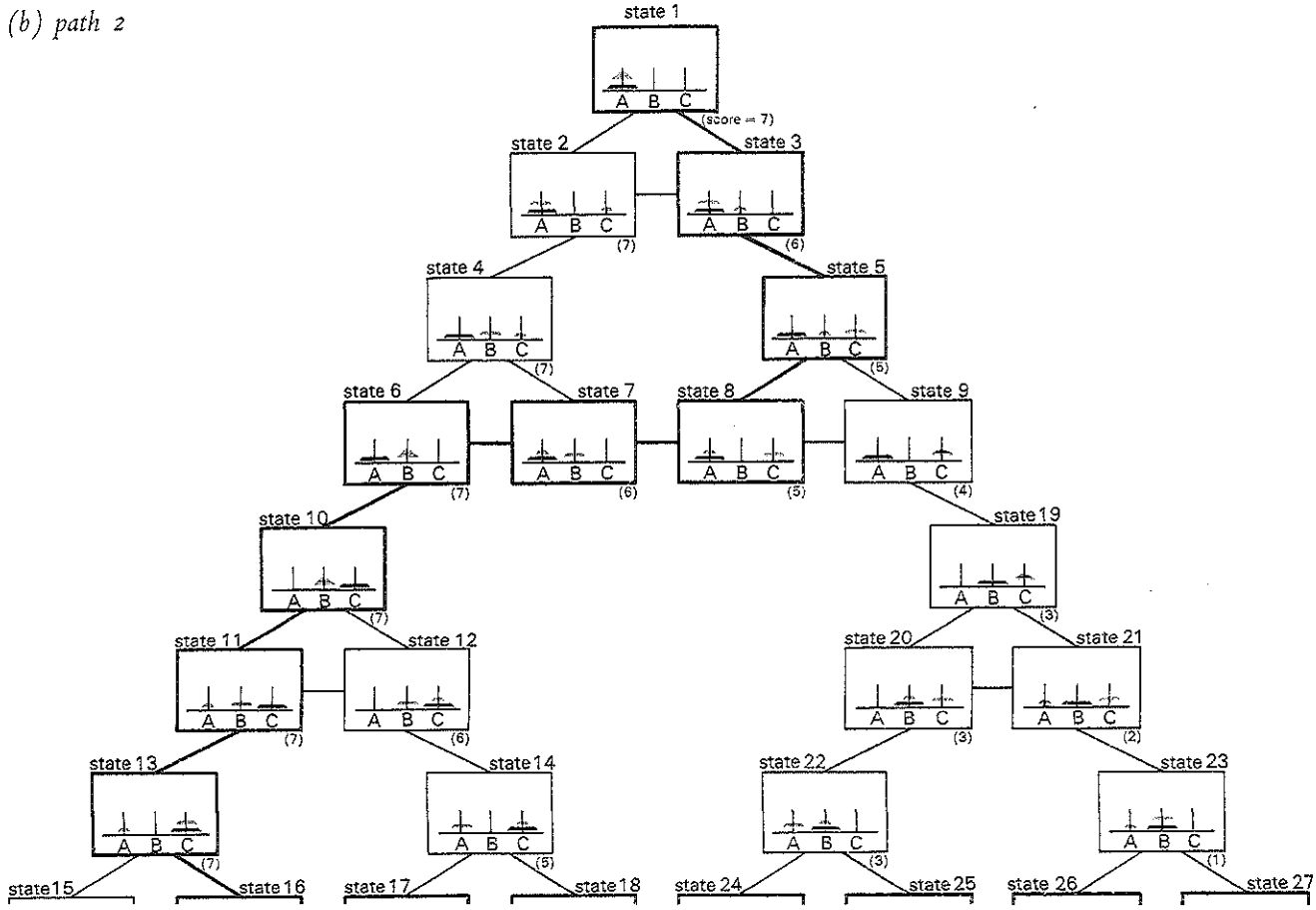
Since the measure on path 1 is non-increasing (always stays the same or decreases) we can describe this path as goal-directed.

Figure 9 Pathways through a three-ring state space

(a) path 1



(b) path 2



SAQ 10 YOUR ANSWER

Is path 2 goal-directed? If not, why not?

SAQ 11 YOUR ANSWER

Consider the same measure and use it for evaluating subgoal-directed paths. Path 1 in Figure 9a goes through three two-ring subproblems.

(a) Is path 1 subgoal directed within each of these subproblems?

(b) Can a path be subgoal-directed for each subproblem entered and not be goal-directed over the entire path? (Check this for path 2 in Figure 9b.)

After a subgoal state is encountered, the path that continues on from a subgoal state can be examined to see if it also happens to exit from the entire subspace of the just-completed subproblem (it might not, of course, if the path doubles back to remain within the same subspace). Intuitively, this means that a subject uses the subgoal as a partial goal of the problem; that is, once arriving at the subgoal, the subject leaves it and goes on to the rest of the problem, rather than staying within the subproblem (this is hypothesis 2).

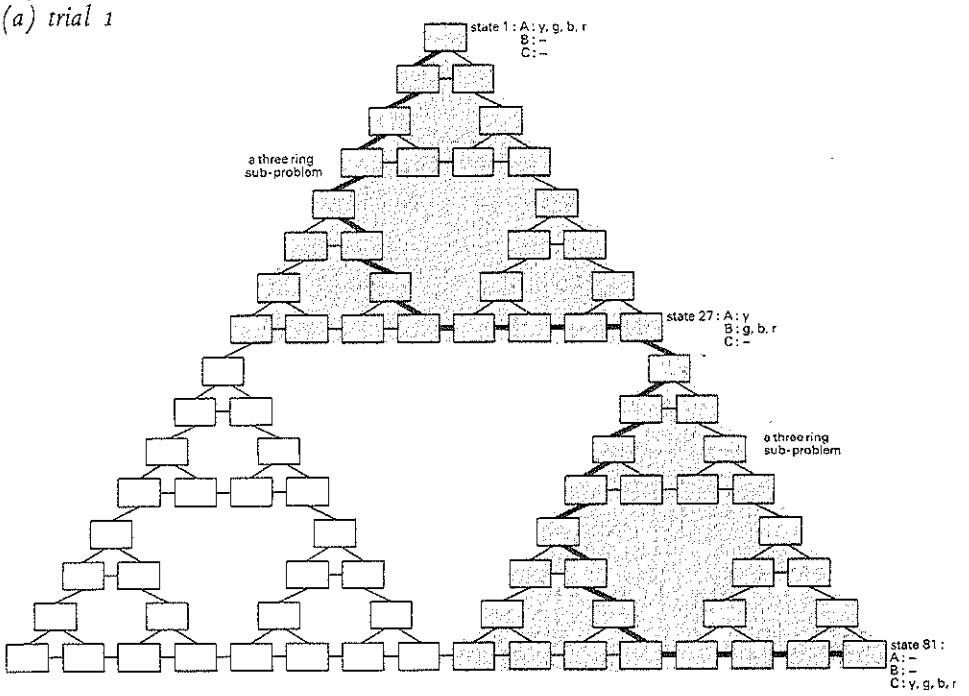
In fact, the five hypotheses listed on p 71 can be analysed in the light of data from subjects solving the Tower of Brahma and Parking Lot problems.

Figure 10 pictures the actual paths through the state space of one adult subject solving the Tower of Brahma problem. It took three trials for the subject to find the shortest solution. All three trials are illustrated in Figures 10a, b and c. Note the following observations.

a All three paths are both goal- and subgoal-directed.

b All three paths always exit completely from a subproblem whenever a subgoal state is exited.

Figure 10 Pathways of one subject through a four-ring state space
(a) trial 1

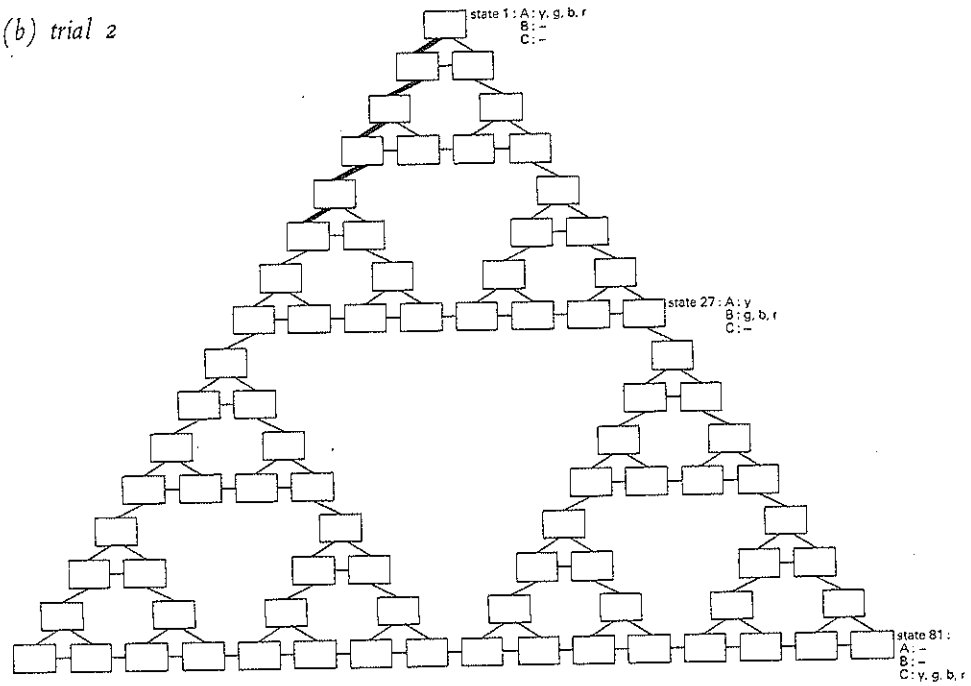


c The first two trials contain seven instances (out of seven attempts) of minimum solutions of two-ring subproblems, while no three-ring subproblems were solved by the shortest path on those first two trials. I refer to this as a two-ring subproblem 'stage' i.e. a stage in the course of the development of the problem solver's expertise on this problem – in this case the subject solves two-ring subproblems perfectly, but still has trouble with three-ring subproblems.

d Trial 1 illustrates two identical (congruent) non-minimum paths through three-ring subproblems.

e Finally, trial 2 is interrupted and trial 3, the solution path, follows. Notice that trial 3 is the symmetric opposite of trial 2.

(b) trial 2



(c) trial 3

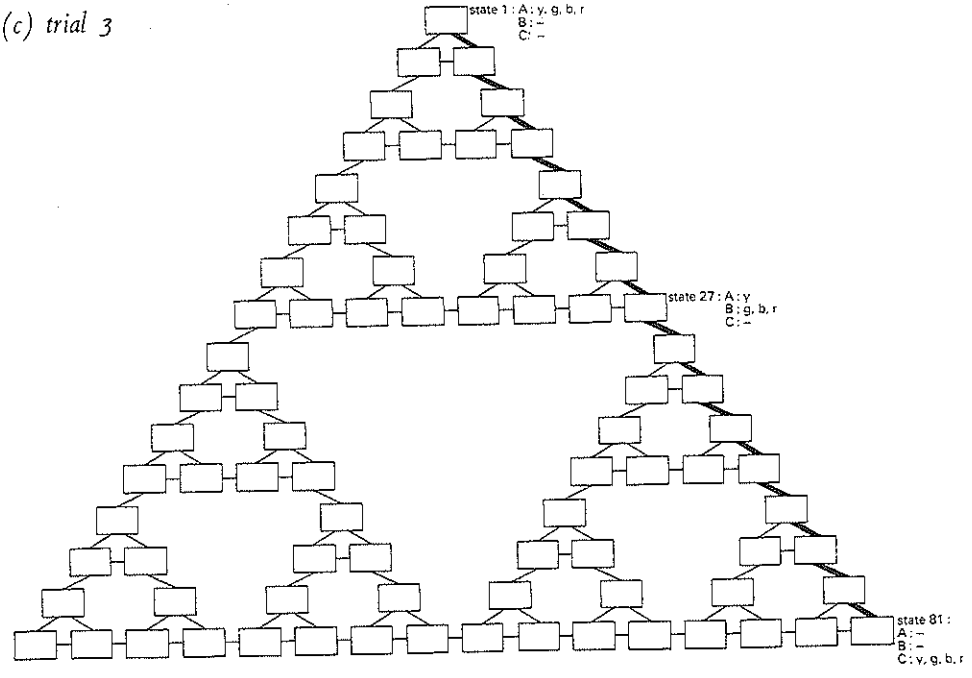


Figure 11 (a – d) pictures the paths (four trials) of another adult subject solving the Parking Lot problem. Note the following observations:

f Two of the paths within the problem are not goal-directed (i.e. trials 1 and 2 – Figures 11a and 11b).

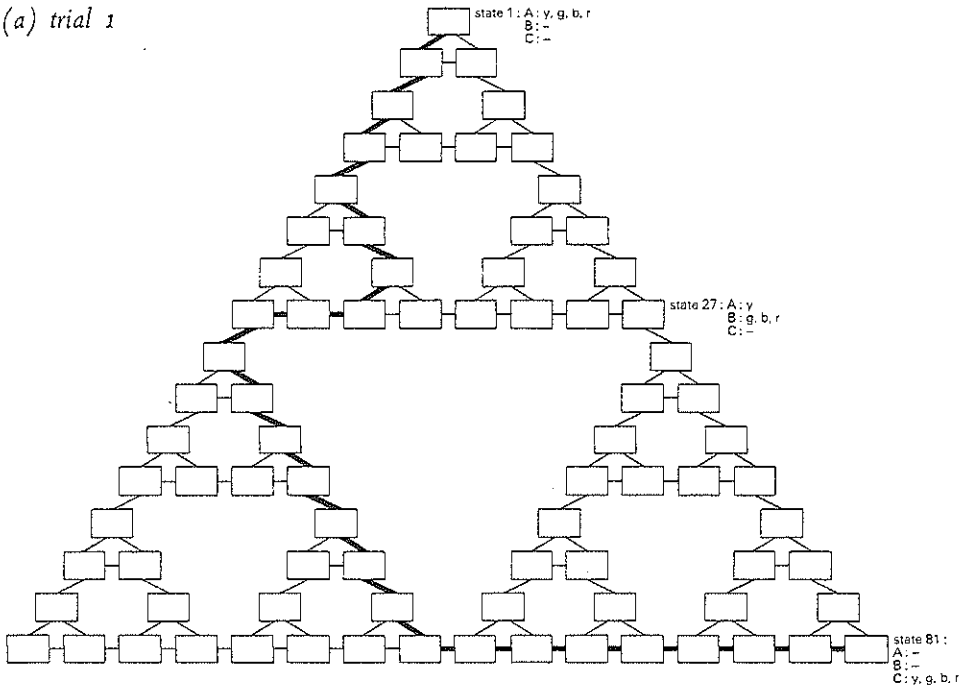
g The paths exit from a subproblem whenever a subgoal state is exited, with one exception: in the shaded two-truck subproblem of trial 2, the move pointed to by the arrow follows the attainment of a subgoal state (striped box), yet *fails* to exit from the subproblem space.

h The subject solves five of the last six three-truck subproblems in the minimum number of steps; this is a three-truck subproblem stage.

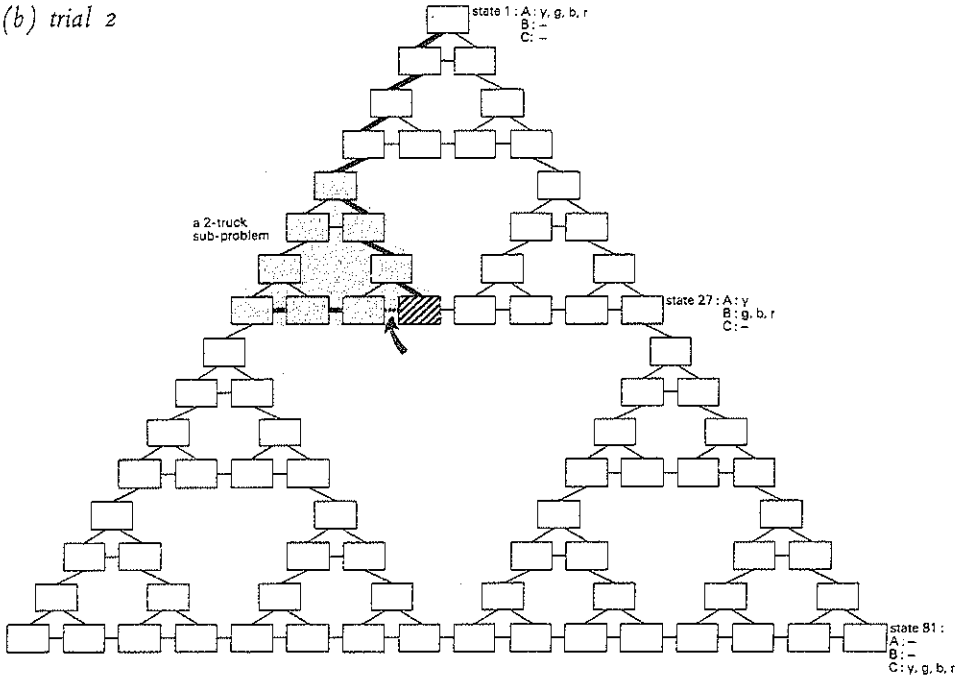
i The third trial is interrupted, and its symmetrically opposite path – which solves the problem – is produced in the fourth trial.

Figure 11 Pathways of one subject through a four-truck state space

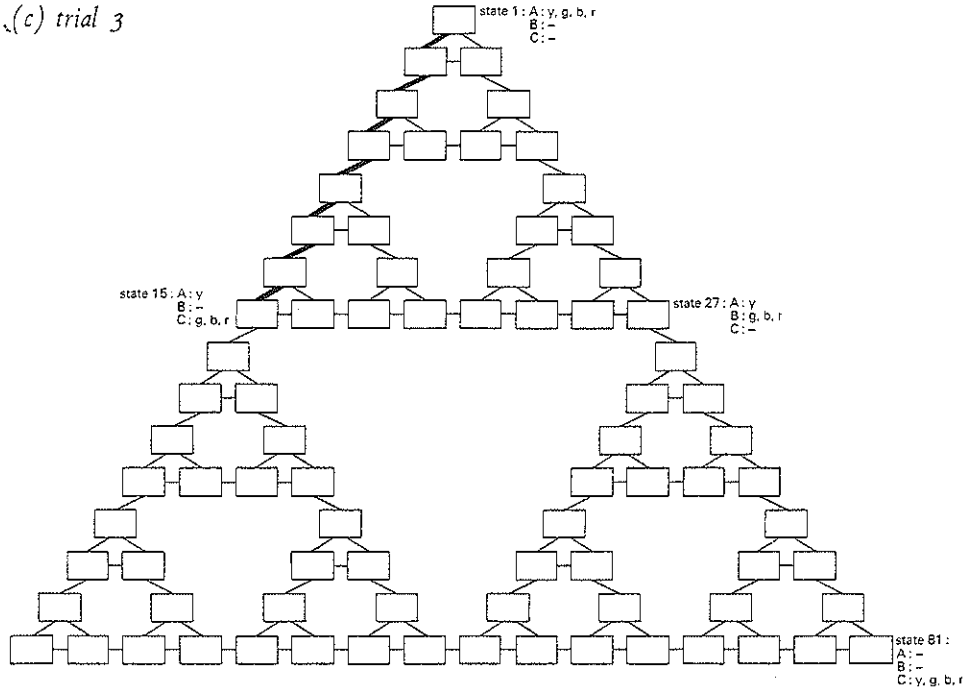
(a) trial 1



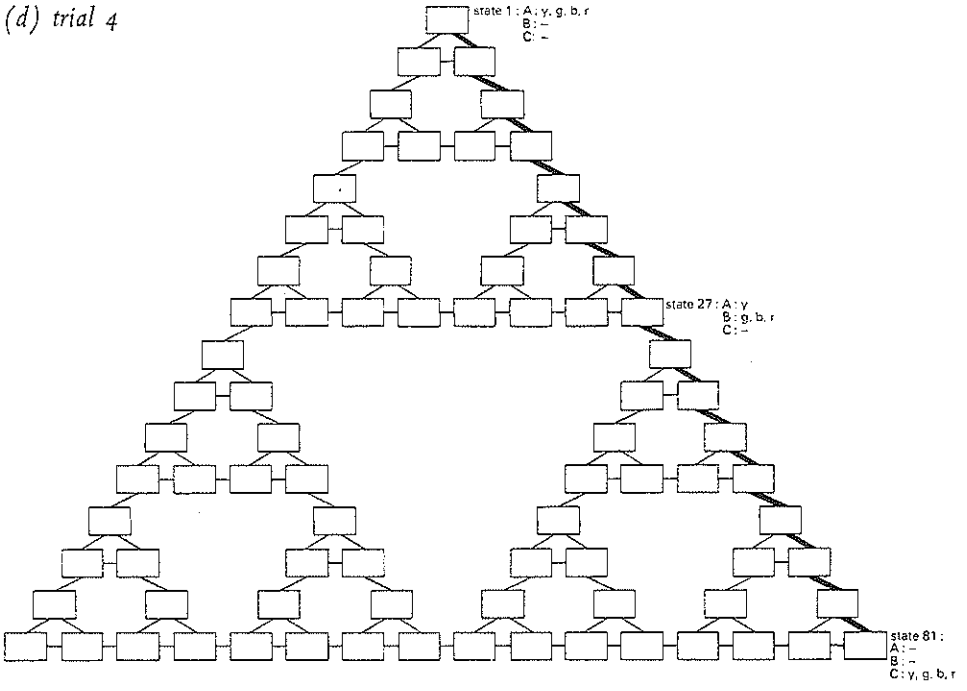
(b) trial 2



(c) trial 3



(d) trial 4



SAQ 12

Ignoring statistical analyses for the moment, compare observations a–i above with the five hypotheses mentioned on p 71, and work out whether the observations support or refute the various hypotheses. You can do this by completing the table:

observation	*relevant hypothesis*	*supported?*
a	1	yes
b		
c		
d		
e		
f		
g		
h		
i		

1.4 Experiments based on state space analysis

The analyses above reflect certain regularities in subjects' behaviour which can be attributed largely to the underlying *structure* of the problems. Other researchers have used state spaces to investigate different aspects of problem solving behaviour. There are many interesting questions which can be asked about the pathway made by a subject through a state space in the course of solving a problem. For instance, how long does a subject spend in making a transition from one state to another? Do transitions between subproblems take noticeably longer? Does experience on a given problem make it easier for subjects to solve new problems which are isomorphic to the original? How do different kinds of task instructions affect the solution time (and pathway) of subjects through a given state space? The highlights of several key studies which raise some of these questions are presented below.

Study: Thomas (1974)

Basic problem used

Missionaries and cannibals (see Units 26–27, section 2).

Variation of problem

Hobbits and orcs used instead. This is the identical problem with missionaries replaced by hobbits and cannibals replaced by orcs.

Purpose of study

To see whether a state space provides the right psychological level of analysis – i.e. do the eleven states of the problem correspond more or less to stages which subjects go through mentally while solving the problem?

Method

One group of subjects first solved the problem starting from the middle (i.e. the 'tricky' state labelled (i) on p 17 of Units 26–27), then solved the *entire* problem. A control group just solved the whole problem once.

Hypothesis

If the state space is the correct level of analysis, then any improved performance by experimental subjects on the entire problem (as compared against the performance of control subjects) should only occur on the *second* half of the problem, because only the states in the second half had been previously encountered.

Measurements used

Number of moves required to find solution; time per move; number of erroneous moves attempted.

Results

Improved performance by the experimental group only occurred on the first half of the problem, while performance on the second half was in fact slightly (but not significantly) worse! The hypothesis was thus rejected. Other analyses showed that subjects respond to states very differently if they ever happen to enter a state a *second* time in the course of a single solution – however, a simple state space analysis would not have predicted this. Also, time measurements revealed three or four major stages in the course of the subjects' solutions, rather than the eleven equally spaced stages (one for each state) predicted by state space analysis.

Implications

(a) State space analysis may serve as a useful underlying framework for carrying out experiments like this one, but the states themselves don't seem to correspond

to the subjects' mental stages, at least on this particular problem.

(b) The difficulty of the 'tricky' state encountered in the problem might be explicable in terms of transitions between more global stages in the solution, rather than simply the subjects thinking that the next move leads to a state which is a blind alley.

Study: Reed, Ernst, and Banerji (1974)

Basic problem used

Missionaries and cannibals ('MC')

Variation of problem

Both missionaries and cannibals, and the following variation were used: 'Jealous Husbands' problem ('JH'), in which missionaries are replaced by husbands and cannibals by wives, with the *additional* constraint that husbands and wives must always be paired, in the sense that a lone wife cannot be left with a husband without *his* wife being present. The state space for this variation is identical to that for MC if only legal moves are considered, but this variation has many more possible illegal moves, because of the pairing rule (a relationship of this type between two state spaces is called *homomorphic*).

Purpose of study

To see whether skill acquired in performing one task could be transferred to an *analogous* (but not identical) task.

Method

Subjects solved two problems – either MC then MC; or JH then JH; or MC then JH; or JH then MC. In addition, some subjects in the last two conditions were explicitly told about the relationship between the two problems.

Hypothesis

Subjects first solving one problem and then the other (i.e. MC then JH or JH then MC) will show improved performance on the second one because of the close correspondence between problem states.

Measurements used

Total time for solution, total number of moves involved, number of illegal moves attempted.

Results

MC then MC: improved performance (fewer moves).
JH then JH: improved performance (fewer moves and faster solution).
MC then JH: no improvement, even when relationship explicitly revealed by experimenter.
JH then MC: improvement *only* when relationship explicitly revealed by experimenter.

Implications

Subjects can clearly capitalize on previous experience of an *identical* problem (e.g. MC then MC again), but to capitalize on experience with an *analogous* problem, they must not only *recognize* the analogous relationship, but must also be able to put this relationship to effective *use* essentially by reducing the total number of operations they would otherwise spend time considering. Hence the improvement for JH then MC (where the recognized relationship makes the MC problem simpler), and the failure of improvement for MC then JH (where the recognized relationship doesn't particularly help, because the new JH problem is still too complex).

Study: Luger and Bauer (1978)

Basic problem used

Tower of Brahma (TOB)

Variation of problem

Tea Ceremony (TC) – this problem is isomorphic to the Tower of Brahma problem. Its verbal description is more complicated, but the physical layout of the problem, as set up by Luger and Bauer, is very similar to the Parking Lot problem you were given on pp 64–5.

Purpose of study

To see whether skill acquired performing one task could be transferred to a second *isomorphic* task, particularly where the tasks have a nice subproblem structure (unlike missionaries and cannibals which can't easily be broken down into subproblems).

Method

Subjects solved either TOB then TC or TC then TOB. They were not told that the two problems were related.

Hypothesis

The isomorphic relationship, combined with the relatively clear structure of the problem and the lack of 'tricky' states, would lead to a transfer of skill, reflected in improved performance on the second task.

Measurements used

Total time to solution; number of states entered; number of illegal moves attempted.

Results

TOB then TC: improved performance
TC then TOB: improved performance
Hypothesis accepted.

Implications

Clear subproblem structure, as well as subjects' ability to experience overall problem symmetry, can enhance transfer of skill from one task to another. The lack of clear subproblem structure could account for the lack of transfer on some of the conditions in the Reed, Ernst, and Banerji study discussed above.

Study: Hayes and Simon (1976)

Basic problem used

Tower of Brahma (three-ring version)

Variation of problem

Several isomorphic variations in which the rules of the puzzle specified legal moves in terms of size changes (e.g. magical shrinking monsters), hereafter referred to as 'change' variations. Other variations stuck to the idea of physically transferring items form one place to another (hereafter referred to as 'transfer' problems).

Purpose of study

To examine the consequences of different verbal formulations of a given problem in order to gain some insight into how problem statements are initially understood by problem solvers.

Method

Subjects solved either

(a) a transfer problem and then a change problem; or

(b) a change problem and then a transfer problem; or

(c) two transfer problems which differed according to whether the monster was the agent (instigator) of the transfer or the patient (victim) of the transfer; or

(d) two change problems which differed according to whether the monster was the agent or patient of the change.

Hypothesis

Even when all the problems used are isomorphic, different task instructions should cause significant differences in problem difficulty (as reflected in solution time), and also in the subject's own internal representation of both the problem states and the legal operators (legal moves).

Measurements used

Time to solve problem; verbal protocols ('thinking out loud').

Results

Transfer problems were solved much more quickly then change problems. Improved performance on the second of each pair of problems was much more evident when the same basic problem was used (i.e. transfer-patient *vs* transfer-agent) than when there was a switch from 'transfer' to 'change' or vice versa. All variations had a strong influence on the representations adopted by the subjects when attempting to solve the problems (as revealed by their verbal protocols).

Implications

Problem structure alone, as reflected in the state space, isn't enough to predict the kinds of difficulties a subject may have in solving a problem. The task of 'understanding' the problem itself, i.e. adopting some internal representation of the states and the operators, will have a drastic effect on the solution process, because some representations may involve much simpler processing operations than others. This research has been followed up with a computer program called UNDERSTAND which simulates the process of building internal representations of a problem which differ for different task instructions (Simon and Hayes 1976).

Progress box one

State space analysis

Aims	To make it possible to identify *regularities* in the behaviour of subjects solving several problems in relation to the *structure* of a problem.
Kind of problems studied	Typically, problems which are either *hierarchically structured* (i.e. have clear subgoals and sub-subgoals, etc.) or have a relatively small search space (small 'tree' of moves), or both. The search space must be small enough for a given researcher to use it to analyse subjects' behaviour. The Tower of Brahma and missionaries and cannibals problems are examples. When a given problem is studied, *isomorphs* of that problem (i.e. problems with identical state spaces) may be studied as well.
Important features	'States' interconnected by 'legal moves'.
Relation to heuristic search (see Units 26–27, section 3)	The notation of states interconnected by legal moves is essentially the same. States in state space analysis may be given *scores* by the researcher in order to determine certain characteristics (e.g. whether the subject keeps moving closer to a goal or subgoal). This is similar to the evaluation function scores of heuristic search, except that heuristic search scores are merely *estimates*, whereas state space analysis scores are after-the-fact scores applied by the researcher, hence they are guaranteed *correct*. Finally, heuristic search is a *problem solving method*, whereas state space analysis is a *descriptive tool* for uncovering regularities in subjects' behaviour.
Advantages	Allows a clear formulation of hypotheses about characteristics of subjects' behaviour, which may then be explicitly and objectively checked by looking at subjects' *paths* through a state space. These hypotheses are concerned with things like the *direction* of the paths (reflecting 'goal-directedness') and the *shape* of the paths (reflecting performance on related subproblems). Also allows the study of *transfer* or learning effects on superficially different problems which have the same underlying structure (which can only be revealed by examining the state space). Finally, this kind of analysis enables us to look in detail at precisely what makes one problem harder than another, even though both have the same underlying state space (i.e. even though they are *isomorphic*).
Disadvantages	Can only be applied to problems whose state space can in practice be completely specified (this won't work for problems like playing chess, where the state space is enormous). Also, this approach tends to concentrate on *what* happened rather than *how* or *why* it happened. That is, it doesn't in itself depict a *method* for solving problems, although it could certainly provide the basis for one.